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LETTER TO THE EDITOR

The one-dimensional Edwards model for long polymer chains

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Abstract. Using the theory of dynamical systems, we investigate the $T\to\infty$ and $g\to 0$ limits of Edwards' probability measure. We prove that there exists at least one limit measure and that this measure is the Wiener measure.

Edwards' model is characterised by a probability measure that differs from the Wiener measure by the presence of a suppressing factor $\exp(-gN(\omega, T))$, where g is a repulsive constant and $N(\omega, T)$ is the number of self-intersections of the path ω. ω is defined on the interval $[0, T]$ and is a subset of $\mathbb{R}^d$ with $d\geqslant 1$. Edwards' model is well known in polymer physics (see [1] and references therein). It is equally interesting because it is related to both ordinary random paths (if $g\to 0$) and self-avoiding paths (if $g\to\infty$) (see [2] and references therein). It is slightly more difficult than the model of ordinary paths but much easier than that of self-avoiding paths. It should be possible to construct the self-avoiding model by taking the $g\to\infty$ limit of Edwards' model. There are, however, only a few mathematical results obtained for it (but see [3, 4]). In particular the $g\to\infty$ and $g\to 0$ limits, as well as the $T\to\infty$ limit, have not been proven yet. We study here the one-dimensional model and present a new approach based on the notion of the dynamical system. This approach is very simple and enables us to prove the existence of at least one $T\to\infty$ limit, obtained by letting g go to zero as T goes to infinity. Furthermore, it should be possible to use the same technique for the treatment of the two-dimensional Edwards model, which is more difficult because of the divergence of $N(\omega, T)$.

We define on Wiener space $\Omega = C([0, T], \mathbb{R})$, with $0<T<\infty$, a probability measure of the form:

$$d\mu(\omega) = Z^{-1}\exp(-gN(\omega, T))\, d\nu(\omega) \tag{1}$$

where g is a positive constant, Z is the normalisation constant and $d\nu$ denotes the Wiener measure. $N(\omega, T)$ represents the number of self-intersections of the Brownian path ω, and is defined as

$$N(\omega, T) = \int_0^T\int_0^T ds\, dt\, \delta[\omega(t)-\omega(s)] \tag{2}$$

where δ denotes the Dirac measure at the origin. Notice that $N(\omega, T)$ is divergent if d is greater than one. If $d=2$ it is, however, possible to give it a sense by renormalisation (see [5]).

Let $\tilde{\omega}(t) = \gamma^{-1}\omega(\gamma^2 t)$. This defines on Wiener space Ω a map S_γ which preserves the Wiener measure ν (this means that $\nu(S_\gamma^{-1}(C)) = \nu(C)$ for every cylinder C). A

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natural question that arises is: what happens to Edwards' measure? First regularise the expression for $N(\omega, T)$

$$N(\omega, T) = (2\pi)^{-1} \int_0^T \int_0^T \mathrm{d}s\, \mathrm{d}t \int_R \mathrm{d}u \exp(\mathrm{i}u(\omega(t) - \omega(s)) \tag{3}$$

which becomes

$$N_\lambda(\omega, T) = (2\pi)^{-1} \int_0^T \int_0^T \mathrm{d}s\, \mathrm{d}t \int_R \mathrm{d}u \exp(\mathrm{i}u(\omega(t) - \omega(s) - \lambda u^2/2)) \tag{4}$$

with $\lambda \geqslant 0$. Therefore,

$$N_\lambda(\omega, T) = (2\pi\lambda)^{-1/2} \int_0^T \int_0^T \mathrm{d}s\, \mathrm{d}t \exp\left(-\frac{(\omega(s) - \omega(t))^2}{2\lambda} \right). \tag{5}$$

A straightforward computation of $N_\lambda(\tilde{\omega}, T)$ leads to the relation

$$N_\lambda(\tilde{\omega}, T) = \gamma^{-3} N_{\lambda\gamma^2}(\omega, \gamma^2 T). \tag{6}$$

Now, using a change of variables theorem, we obtain:

$$\mu_{g,T}(S_\gamma^{-1}(C)) = \mu_{\gamma^3 g, T/\gamma^2}(C) \tag{7}$$

for every cylinder C and thus for every Borel set A (from now on we write down μ with its two parameters g and T).

This means that Edwards' measure is not invariant. One can say it is 'covariant' with respect to S_γ in the sense that the image measure of $\mu_{g,T}$ under S_γ is still a measure of Edwards type but with new parameters $g' = \gamma^3 g$ and $T' = T/\gamma^2$.

Remark. If $A = \Omega$ then the above formula implies that

$$Z(\gamma) \equiv \int_\Omega \exp(-g/\gamma^3 N(\omega, \gamma^2 T)) = \int_\Omega \exp(-gN(\omega, T)) \equiv Z. \tag{8}$$

The normalisation factor is thus independent of γ.

Formula (7) suggests that we allow g go to zero as T goes to infinity so as to use the covariance of the measure. In other words, starting from a given μ_{g_0, T_0} we follow, in the (g, T) plane, the hyperbola defined by $g^{2/3} T = g_0^{2/3} T_0$. This $T \to \infty$ limit can be viewed as an infinite-volume $g \to 0$ limit.

We now consider the dynamical system canonically associated with Wiener space Ω. We define $S = \{S_t, t \geqslant 0\}$ by

$$S_t \omega(u) = \exp(t/2)\omega(\exp(-t)u). \tag{9}$$

We recover S_γ by putting $\gamma = \exp(-t/2)$. S preserves ν and is exact [6]; this means that if $\nu(A) > 0$ then $\lim_{t\to\infty} \nu(S_t(A)) = 1$ for every Borel set A. Notice that exactness implies ergodicity [6].

We associate with S the semigroup of Frobenius-Perron operators $P = \{P_t, t \geqslant 0\}$ defined by the relation

$$\int_A P_t f(\omega) \nu(\mathrm{d}\omega) = \int_{S_t^{-1}(A)} f(\omega)\nu(\mathrm{d}\omega). \tag{10}$$

Then (7) and (10) imply that

$$P_t \exp(-gN(\omega, T)) = \exp(-\gamma^3 g N(\omega, T/\gamma^2)) \text{ a.s.} \tag{11}$$

where $t = -2\ln\gamma$. The limit $T \to \infty$ corresponds now to the limit $\gamma \to 0$ which, in turn, is related to the $t \to \infty$ limit of the semigroup P.

Remark. t is a continuous variable, but we can imagine that t takes only rational values, which is not a restriction at all.

One can show that P is asymptotically stable [5]. Otherwise stated, there exists a positive function ρ such that:

(i) $\int_\Omega \rho \, d\nu = 1$ and $P_t \rho = \rho$ for every $t \geq 0$

(ii) $\lim_{t\to\infty} P_t f = \rho$ in $L^1(d\nu)$ if f is positive and $\int_\Omega f \, d\nu = 1$.

Thus, by (ii),

$$\rho \equiv \lim_{\gamma \to 0} \frac{\exp(-\gamma^3 g N(\omega, T/\gamma^2))}{Z(\gamma)} \tag{12}$$

exists in $L^1(d\nu)$. Now the invariance of the Wiener measure and ergodicity imply that $\rho = 1$! This means that $Z^{-1} \exp(-gN(\omega, T))$ converges in L^1 to 1 when T goes to infinity and g to zero, provided that g and T are related as (11). So the limit measure obtained above is just the Wiener measure! This is not surprising if, as explained before, this limit is interpreted as an infinite-volume $g \to 0$ limit. Notice, however, that this result does not exclude convergence to other limits if $g \to 0$ and $T \to \infty$ following different routes in the (g, T) plane.

In two dimensions the problem is much more complicated because of the divergence of $N(\omega, T)$. It is necessary to renormalise $N(\omega, T)$ and to change the parametrisation γ. The treatment of the two-dimensional Edwards model is the object of a forthcoming paper.

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